

- 1.1: 2  
 1.2:  $e^2$   
 1.3: (-2,3)  
 1.4:  $-\frac{2}{e}$   
 1.5: 2  
 1.6: 1  
 1.7:  $\frac{1}{3} \text{ArcTan}[3e^x] + C$   
 1.8: 4  
 1.9: 1  
 1.10: 0  
 1.11:  $\begin{pmatrix} -bc+ad & 0 \\ 0 & -bc+ad \end{pmatrix}$   
 1.12: 2

2.1  $\lim_{x \rightarrow \pi} \frac{\tan x - \sin x}{\pi - x}$  罗比达法则  $\lim_{x \rightarrow \pi} \frac{\sec^2(x) - \cos(x)}{-1} = -2$

2.2  $f'(x) = -2x + x^2$  有两个零点, 即  $x_1 = 0, x_2 = 2$

	$(-\infty, 0)$	0	$(0, 2)$	2	$(2, +\infty)$
$f'(x)$	+	0	-	0	+
$f(x)$	↗	1/3	↘	-1	↗

所以单增区间:  $(-\infty, 0]$  和  $[2, +\infty)$ , 单减区间  $[0, 2]$ , 在  $x=0$  时取得极大值  $1/3$ ,  $x=2$  时取得极小值  $-1$ .

2.3  $\int \frac{\sqrt{x}}{1-x} dx \stackrel{x=t^2}{=} \int \frac{2t^2}{1-t^2} dt = \int \frac{2t^2-2+2}{1-t^2} dt = \int (\frac{2}{1-t^2} - 2) dt = -2t + \int (\frac{1}{1-t} + \frac{1}{1+t}) dt$   
 $= -2t - \ln |1-t| + \ln |1+t| + C = -2\sqrt{x} - \ln |1-\sqrt{x}| + \ln |1+\sqrt{x}| + C$   
 $= -2\sqrt{x} + \ln \left| \frac{1+\sqrt{x}}{1-\sqrt{x}} \right| + C$

2.4  $\int_0^{\pi/4} \cos \sqrt{x} dx \stackrel{x=t^2}{=} \int_0^{\pi/2} 2t \cos t dt = \int_0^{\pi/2} 2t d(\sin t)$   
 $= (2t \sin t) \Big|_0^{\pi/2} - \int_0^{\pi/2} 2 \sin t dt$   
 $= \pi + 2(\cos t) \Big|_0^{\pi/2} = \pi - 2$

2.5 由题意  $A = g(0) = \lim_{x \rightarrow 0} g(x) = \lim_{x \rightarrow 0} \frac{f(x)+2\sin x}{x} = \lim_{x \rightarrow 0} \frac{f(x)-f(0)+2\sin x}{x} = f'(0) + 2 = 1+2=3$

2.6  $AX + I = A^2 + X \Leftrightarrow (A - I)X = A^2 - I = (A - I)(A + I)$

易知  $A - I = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$  可逆, 故原方程的解为:

$$X = A + I = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{pmatrix}$$

2.7 增广矩阵:  $\begin{pmatrix} 2 & -1 & 1 & -1 & 2 \\ 1 & -2 & 2 & 1 & 4 \\ 1 & -3 & 4 & 3 & 8 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 1 & -2 & 2 & 1 & 4 \\ 2 & -1 & 1 & -1 & 2 \\ 1 & -3 & 4 & 3 & 8 \end{pmatrix}$

$$\xrightarrow{R_2 - 2R_1} \begin{pmatrix} 1 & -2 & 2 & 1 & 4 \\ 0 & 3 & -3 & -3 & -6 \\ 1 & -3 & 4 & 3 & 8 \end{pmatrix} \xrightarrow{R_3 - R_1} \begin{pmatrix} 1 & -2 & 2 & 1 & 4 \\ 0 & 3 & -3 & -3 & -6 \\ 0 & -1 & 2 & 2 & 4 \end{pmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{pmatrix} 1 & -2 & 2 & 1 & 4 \\ 0 & -1 & 2 & 2 & 4 \\ 0 & 3 & -3 & -3 & -6 \end{pmatrix}$$

$$\begin{aligned}
&\xrightarrow{-1R_2} \begin{pmatrix} 1 & -2 & 2 & 1 & 4 \\ 0 & 1 & -2 & -2 & -4 \\ 0 & 3 & -3 & -3 & -6 \end{pmatrix} \xrightarrow{R_1+2R_2} \begin{pmatrix} 1 & 0 & -2 & -3 & -4 \\ 0 & 1 & -2 & -2 & -4 \\ 0 & 3 & -3 & -3 & -6 \end{pmatrix} \xrightarrow{R_3-3R_2} \begin{pmatrix} 1 & 0 & -2 & -3 & -4 \\ 0 & 1 & -2 & -2 & -4 \\ 0 & 0 & 3 & 3 & 6 \end{pmatrix} \\
&\xrightarrow{\frac{1}{3}R_3} \begin{pmatrix} 1 & 0 & -2 & -3 & -4 \\ 0 & 1 & -2 & -2 & -4 \\ 0 & 0 & 1 & 1 & 2 \end{pmatrix} \xrightarrow{R_1+2R_3} \begin{pmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & -2 & -2 & -4 \\ 0 & 0 & 1 & 1 & 2 \end{pmatrix} \xrightarrow{R_2+2R_3} \begin{pmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 2 \end{pmatrix}
\end{aligned}$$

所以通解为

$$x_1 = c, \quad x_2 = 0, \quad x_3 = 2 - c$$

$$3.1 \quad f(t) = \lim_{x \rightarrow \infty} t \left(1 + \frac{1}{x}\right)^{2tx} = \lim_{x \rightarrow \infty} t \left(1 + \frac{1}{x}\right)^{2t} = t e^{2t}$$

$$f'(t) = (t e^{2t})' = e^{2t} + 2t e^{2t} = e^{2t}(1 + 2t)$$

3.2

$r(A) + r(B) - r(A+B)$	A	$r(A)$	B	$r(B)$	A+B	$r(A+B)$
0	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	0	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	0	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	0
1	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$	1	$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$	1	$\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$	1
2	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$	1	$\begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}$	1	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	0
3	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	2	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	2	$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	1
4	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	2	$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	2	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	0